

Increased fracture toughness of ceramics by energy-dissipative mechanisms

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A theoretical model for the fracture toughness of ceramics is developed which takes into account such energy-dissipative mechanisms as stress-induced microcracking or phase transformation. To establish the general fracture criterion, a Griffith-type energy balance is employed. This energy balance comprises the elastic energy, the fracture surface work consumed in the process zone at the crack tip, the energy dissipated in the dissipation zone and the energy stored by residual stresses. Stress-induced microcracking is considered in more detail. An expression for the dependence of the fracture toughness on the density of microcracks, the amount of residual stresses caused by thermal expansion mismatch between the ceramic matrix and small particles embedded in it and the volume fraction of these particles is derived. The final results are used to state conditions necessary for the fracture toughness to be increased. The theory agrees well with experimental results taken from literature (alumina with zirconia particles).

1. Introduction

It is well known that ceramics are brittle materials. As a result, failure in ceramics is often catastrophic and this is the main draw-back of most ceramics; because of this much scientific work has been performed in order to develop strong, tough ceramics. The different mechanisms which could be used to achieve this goal have been reviewed, for example, by Evans *et al.* [1] and Lange [2].

In this paper energy-dissipative mechanisms are dealt with. Such a mechanism acting in front of a crack tip is capable of absorbing energy that is otherwise available for crack propagation. Two examples of this may be found in the ceramic literature: stress-induced microcracking and stress-induced phase transformation.

The original idea concerning microcracking as an energy-absorption mechanism was first formulated by Glucklich and Cohen [3] who considered a concrete-type material. The same idea was used by Gupta [4] and Green *et al.* [5] in an attempt to explain the resistance to thermal shock, and the fracture behaviour, of partially-stabilized zirconia (PSZ). Hoagland *et al.* [6] examined the influence of microstructure on fracture propagation in rock.

They found that microcracking may have a considerable influence on the resistance to crack propagation. More recently, Claussen [7, 8] has used the idea of stress-induced microcracking to explain the increased fracture toughness observed if unstabilized zirconia is added to alumina.

Energy absorption by stress-induced phase transformation was originally utilized to increase the toughness of certain steels (so-called TRIP steels). Concerning ceramics, Garvie *et al.* [9] suggested that the high toughness of calcia-PSZ ceramics containing tetragonal zirconia resulted from absorption of energy during the martensitic transformation of tetragonal particles to monoclinic ones. Such PSZ ceramics, stabilized by yttria, calcia or magnesia, were also considered by Porter and Heuer [10], Gupta *et al.* [11, 12] and Lange [2, 13].

Recently some theoretical work concerning energy-dissipative mechanisms has been published. There exist different approaches. Let us first discuss those which are based on an energy concept. This concept is inspired by the analogy between the energy-dissipative mechanisms considered here and energy dissipation by plastic deformation. In

this way Antolovich [14] calculated the energy dissipated by phase transformation in TRIP steels. Gupta *et al.* [12] and Lange [2, 13] used the results of Antolovich in an attempt to determine the fracture toughness of PSZ ceramics where stress-induced phase transformation occurs. Recently, Porter *et al.* [15] published a paper which presents a detailed examination of energy absorption by phase transformation in PSZ ceramics. In the case of stress-induced microcracking, the energetic approach is also very suitable for studying the stability of a macroscopic crack i.e. determining the fracture toughness. This has been done by the present authors [16–18].

Contrary to the energetic approach, other calculations may be found in literature which are not based on a strict stability analysis. Therefore some ambiguities are involved in the definition of the critical point which governs the fracture toughness. For instance Evans *et al.* [1] have assumed that instability occurs if the density of stress-induced microcracks reaches a certain critical value in the vicinity of the tip of a macrocrack. That means they neglect any Griffith-type instability prior to this point. The investigations of Buresch [19–21] represent a more detailed examination of stress-induced microcracking in alumina ceramics. Although some energy considerations are involved, the calculation of the critical point, however, seems not to be quite correct since it is based on Neuber's two-parameter fracture criterion which is not equivalent to a stability analysis for a macrocrack interacting with a large number of microcracks.

The above discussion has lent support to the opinion that only a strict stability analysis based on energy methods [12, 13, 17] can form a basis for theoretical considerations. Until now, however, an important feature inherent to most ceramics has been neglected: the energy stored in residual stress fields. Although it is well known that, for instance, thermal stresses markedly affect the mechanical properties of ceramics (cf. Lange [2]), theories concerning energy dissipative mechanisms which include such residual stresses are not yet available. The present paper is intended to make a contribution to this area of difficulty; a theoretical model will be developed which will describe the influence of energy-dissipative pro-

cesses and residual stresses on the fracture toughness of ceramics. At first the general fracture criterion will be presented. Then we shall consider stress-induced microcracking in more detail as an example of an energy-dissipative mechanism.

2. Energy criterion for crack propagation

In this section a general expression for the fracture toughness, obtained by making a Griffith-type energy balance for crack propagation, is derived. In doing so account has to be taken of the release of elastic energy, the fracture surface work, the dissipated energy and the change of energy stored in residual stress fields.

Consider a ceramic sample of unit thickness with a macroscopic plane crack of length l . When this sample is loaded, energy dissipation may occur in the vicinity of the tip of the macrocrack while in the remaining part of the sample the material behaves elastically (Fig. 1). The characteristic length, r_D , of the region where energy is dissipated depends on the applied load (it will be determined later). We term this region the "dissipation zone".* In this zone elastic energy is converted into heat, fracture surface energy of microcracks or energy connected with acoustic emissions or phase transformation. This amount of energy is denoted by U_D . Simultaneously the residual stresses change, and hence the stored energy, say U_s , must be included in the energy balance.

We have to distinguish another zone. This is the region just ahead of the tip of the macrocrack where direct interaction processes are important. For instance coalescence with microcracks may occur in this zone when the macrocrack advances. We call this region the "process zone". The corre-

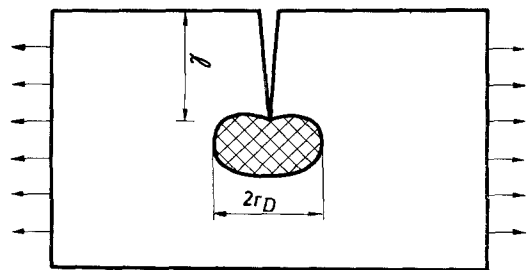


Figure 1 Dissipation zone at the tip of a macrocrack.

*In the literature different names may be found for this e.g. "transformation zone", "process zone", "microcrack zone", "quasi-plastic zone". We prefer the general term "dissipation zone".

sponding characteristic parameter is represented by γ , the energy which is effectively consumed per unit area in creating a fracture surface at the tip of the macrocrack. γ may be less than or greater than the true fracture surface energy, γ_0 , of the material (e.g. coalescence with microcracks or branching may occur which will influence the effective fracture surface energy).

The aim of this paper is to calculate the macroscopic fracture toughness of the ceramic material. That means that the stability of the macrocrack must be examined. To this end an attempt could be made to solve the elastic problem for the macrocrack interacting with the elements of the dissipation zone (e.g. this zone may contain a large number of microcracks). Using this solution we could impose a small variation on the length of the macrocrack, thus investigating the stability. This is, however, an insoluble problem because the exact locations of the microcracks are not known (to say nothing of the mathematical difficulties). In addition, some ambiguities are involved in the definition of crack advance because homogeneity in the direction parallel to the front of the macrocrack is no longer maintained. It is actually not necessary to consider all the microscopic processes in full detail, because the fracture toughness, which is obtained from an experiment on the macroscopic level, is governed by the average action of the local processes. That means, for the first step, it is appropriate to look upon the dissipation zone as a homogeneous region which can be described by average quantities as for example "dissipated energy per unit volume". In addition it can be assumed that this zone is small in size compared with the geometric dimensions of the sample such as notch length, unnotched specimen width and so on. This is equivalent to saying that the variation of the length of the macrocrack is large compared with the characteristic microscopic dimensions but still small compared with the length of the macrocrack itself. In ductile fracture mechanics, the situation envisioned has been termed "small-scale yielding". Thus the problem of investigating the stability of the macrocrack has been considerably simplified since merely some average properties of the dissipation zone enter into the stability analysis. Only in the second step (Section 3) will we relate the average properties to the microscopic features of the ceramic material.

Now let us examine the processes connected

with the variation dI of the length of the macrocrack. Within the small-scale yielding approach the release of elastic energy is equal to the loss of potential energy, $-dP$, such that

$$-dP = G dl, \quad (1)$$

where G is the ordinary elastic energy release rate. In order to set up the balance of energy, we must compare the available energy in Equation 1 with the energy, $d\Sigma$, necessary for crack extension

$$d\Sigma = 2\gamma dl + dU_D + dU_s. \quad (2)$$

In this expression $2\gamma dl$ is the fracture surface energy consumed in the process zone, dU_D denotes the energy dissipated when the dissipation zone is shifted according to the extension of the macrocrack and dU_s is the change of stored energy (normally, dU_s is negative since stored energy is released).

The critical value of the energy release rate, i.e. the fracture toughness G_c , is reached if the available energy in Equation 1 is equal to the necessary energy in Equation 2. This is identical to the statement that the variation of the total energy vanishes

$$dP + d\Sigma = 0. \quad (3)$$

Making use of Equations 1 and 2

$$G_c = 2\gamma + \frac{dU_D}{dl} + \frac{dU_s}{dl} \quad (4)$$

is obtained. Here specific values for U_D and U_s may be introduced. Let η_D and η_s denote the energy dissipated and stored, respectively, per unit volume. In general, η_D and η_s may be functions of position, hence U_D and U_s can be written as

$$U_D = \int \eta_D(\vec{r}) dV, \quad U_s = \int \eta_s(\vec{r}) dV. \quad (5)$$

In this paper the transition zone which may exist at the boundary of the dissipation zone is neglected. That means it is approximately assumed that $\eta_D(\vec{r})$ changes from 0 outside the dissipation zone to a constant value η_D inside the zone.

For the present let us look upon η_D as a given material parameter which depends only on the features of the particular dissipation process to be considered. The same holds for the specific stored energy $\eta_s(\vec{r})$ which changes from η_{s0} outside to η_s inside the dissipation zone (η_{s0} denotes the energy which is stored in the material by residual stresses before loading).

Now the increments dU_D and dU_s may be

easily calculated by multiplying η_D and $(\eta_s - \eta_{s0})$, respectively, by the newly-created volume of the dissipation zone. In our case of a small dissipation zone this volume is given by $2r_D dl$ since the whole dissipation zone is shifted by an amount dl without changing its shape (cf. Fig. 1). Thus we obtain

$$\begin{aligned} dU_D &= 2r_D \eta_D dl; \\ dU_s &= 2r_D (\eta_s - \eta_{s0}) dl. \end{aligned} \quad (6)$$

With the help of these results, Equation 4 may be rewritten as

$$G_c = 2\gamma + 2r_D (\eta_D + \eta_s - \eta_{s0}).^* \quad (7)$$

The size of the dissipation zone r_D can be calculated in an analogous manner to the case of plastic zones. Since it is sufficient to consider small-scale yielding, r_D is given by

$$r_D = \alpha \frac{G_c E_0}{\sigma_c^2}. \quad (8)$$

σ_c is a characteristic stress at which energy dissipation starts, E_0 denotes Young's modulus of the material and α is a constant which depends on the features of the dissipation process. A reasonable choice is

$$\alpha = \frac{1}{3} \quad (9)$$

which is in accordance with results obtained for plastic zones (e.g. Rice [23]) or microcrack zones (Hoagland *et al.* [24]). From Equations 7 and 8 we finally obtain

$$\frac{G_c}{2\gamma_0} = \frac{\gamma/\gamma_0}{1 - \alpha \left[\frac{\eta_D + \eta_s - \eta_{s0}}{\sigma_c^2 / (2E_0)} \right]}. \quad (10)$$

In this formula the fracture toughness has been normalized by $2\gamma_0$ which is the ordinary toughness of the material if there is no additional energy dissipation.

Equation 10 represents a general expression for the fracture toughness provided that η_D , η_s and η_{s0} are determined by material parameters and do not depend on the applied load or sample geometry. However, a final conclusion with regard to an increase in G_c cannot be drawn from Equation 10 since, for example, the formation of microcracks and residual stresses may have a quite different influence on γ/γ_0 , η_D , $(\eta_s - \eta_{s0})$ and σ_c . There-

fore, in the next section, these parameters will be related to microstructural properties of the ceramic material in order to decide whether the toughness increases or not.

3. Energy dissipation by formation of microcracks

Up to now the fracture process has only been considered from a general point of view. Now we come to the details of the dissipation process. Let us look at a ceramic material consisting of a matrix with small particles embedded in it (for example one may think of alumina with zirconia particles). Suppose that stable microcracking appears in the matrix when the stress reaches a certain critical value. Microcracking means that a large number of small microcracks are formed starting from pre-existing flaws. For the sake of simplicity these microcracks are assumed to be penny-shaped with a radius a . The existence of this (finite) length a presupposes that the microcracks are stopped at some obstacles, such as grain boundaries, matrix-particle interfaces or by localized stress fields. This is an essential presupposition since otherwise the formation of the first microcrack leads to instability of the whole sample.

Let \mathcal{N} denote the number of microcracks per unit matrix volume. Then a dimensionless density of microcracks ρ may be defined by

$$\rho = \mathcal{N}(2a)^3. \quad (11)$$

In this paper ρ is regarded as a given material-dependent constant which is determined by the microstructure of the ceramic material in question.

The formation of microcracks causes energy dissipation, since elastic energy is converted into fracture surface energy of microcracks, energy of elastic waves and other irreversible transformations of the microstructure. However, in addition to that dissipated energy, the energy stored in residual stress fields enters into the general energy balance also (cf. Equation 4). This is an essential point, because we are going to investigate ceramic composites, in which residual stresses may act in two different ways. On the one hand, simultaneously with the formation of microcracks, part of the stored energy is released. On the other hand,

*If a transition zone at the boundary of the dissipation zone is taken into account, then η_D , η_s and η_{s0} have to be replaced by certain average values. A strict examination of this problem using finite element methods will be given in [22].

residual stresses cause a decrease in the critical stress σ_c , which is equivalent to a large dissipation zone (cf. Equation 8). The former mechanism diminishes the effect of energy dissipation whereas the latter one enhances it. In the following section the various quantities are considered in more detail.

3.1. Calculation of critical stress

Firstly, a model describing the elastic behaviour of the composite is introduced. Since formulae which are as simple as possible are preferred, the widely known composite—sphere model of Hashin [25] is adopted. This model is very suitable for this case, i.e. for investigating a composite consisting of isolated particles embedded in a matrix. The essential features of the composite—sphere model may be described as follows. The composite is thought to be composed of so-called composite spheres of different sizes. The radii of these spheres are chosen in such a manner that the volume of the body can be completely filled. Every composite sphere, in turn, is composed of a spherical particle encased by matrix material. Let R and R' denote the radius of the particle and the outer radius of the sphere, respectively. Then

$$v = (R/R')^3 \quad (12)$$

is the volume fraction of the particle of an individual composite sphere. It is always possible to choose values for R such that the "individual" volume fraction (Equation 12) is the same for all spheres and, moreover, v is equal to the given volume fraction of particles of the composite. With these assumptions it can be shown that the elastic properties of the composite are equivalent to those of an individual composite sphere [25].

This paper is concerned with the matrix stress which arises if the matrix and the particle undergo different stress-free strains which may be due to a difference in the thermal expansion coefficients as well as to a phase transformation of the particles. Let $(\epsilon_{ij}^T)^p$ and $(\epsilon_{ij}^T)^m$ be the stress-free strains of the particles and the matrix, respectively. The case is restricted to isotropic transformation strains. Hence

$$\Delta\epsilon_{ij}^T = (\epsilon_{ij}^T)^p - (\epsilon_{ij}^T)^m = \epsilon^T \delta_{ij} \quad (13)$$

can be written. Then, within the composite—sphere model, the complete solution for the elastic stress fields in the composite can be derived.

In this paper, however, only some particular

results are given. With regard to the state prior to microcracking, stress concentrations due to different elastic moduli compared with those due to different stress-free strains since the latter seems more important may be neglected. That means, that it is possible to assume that the elastic properties of particles and matrix are approximately identical (before microcracking):

$$\begin{aligned} E^p &= E^m = E_0; \\ \nu^p &= \nu^m = \nu_0. \end{aligned} \quad (14)$$

Here, E and ν depict Young's modulus and Poisson's ratio, and the subscript "0" refers to the composite elastic moduli. Then the matrix tangential stress in the vicinity of a particle is given by

$$\begin{aligned} \sigma_t^m(r) &= \frac{2}{3(1-\nu_0)} E_0 \left[\frac{1}{2} \left(\frac{R}{r} \right)^3 + \nu \right] \epsilon^T, \\ R &\leq r \leq R' \end{aligned} \quad (15)$$

where r denotes the distance from the centre of the particle.

Later, in the more general case, the average matrix stress is also needed if there is a difference between Young's moduli of the particle and the matrix

$$\begin{aligned} E^p &= E_0 \quad E^m = E; \\ \nu^p &= \nu^m = \nu_0 = 0.2. \end{aligned} \quad (16)$$

The value $\nu_0 = 0.2$ has been chosen because it provides a lucid result. This is justified since there is not strong dependence on Poisson's ratio. Then the mean value of the matrix stress is obtained

$$\bar{\sigma}_{ij}^m = \frac{5}{3} \left\{ \frac{vE}{[(1 + E/E_0) + v(1 - E/E_0)]} \right\} \epsilon^T \delta_{ij}. \quad (17)$$

The following model describing the formation of microcracks may now be established. Microcracking is assumed to start if the maximum of the total tangential stress reaches the critical value σ_{mc} . Since the total stress is a simple superposition of applied and residual stresses given by Equation 15, we have

$$\sigma_{mc} = \sigma_c + \sigma_t^m(R) = \sigma_c + \frac{2}{3(1-\nu_0)} E_0 \left(\frac{1}{2} + \nu \right) \epsilon^T,$$

where, according to Equation 8, σ_c denotes the critical value of the applied stress necessary to induce microcracking. If this equation is solved for σ_c

$$\frac{\sigma_c}{\sigma_{mc}} = 1 - \frac{5}{12}(1 + 2\nu)\chi \quad (18)$$

is obtained, for the restricted case of $\nu_0 = 0.2$. The dimensionless parameter χ is a combination of material properties

$$\chi = \frac{E_0 \epsilon^T}{\sigma_{mc}}. \quad (19)$$

Hence, χ may be looked upon as a measure of the residual stresses.

Let us consider the case when the ceramic composite has been cooled from the fabrication temperature and the particles have undergone a phase transformation at a certain temperature connected with a linear expansion $\Delta\epsilon^P$. In addition, take into account the different thermal expansion coefficients α^P and α^m of the particles and the matrix. Let T_1 denote the temperature below which stresses are no longer relaxed. T_0 is the ambient temperature. Then for ϵ^T

$$\epsilon^T = (\alpha^m - \alpha^P)(T_1 - T_0) + \Delta\epsilon^P. \quad (20)$$

If we consider the tetragonal \rightarrow monoclinic transformation of zirconia particles, Equations 13 and 20 must be seen as approximations since that transformation is not perfectly isotropic. However, it does not seem to be justified to consider this anisotropy in full detail because we have already made some approximations of the same level.

Finally, it should be mentioned that, in the literature, although one may find expressions for the tangential stress (Equation 15) which only take into account the case $E^P \neq E^m$, there is usually another restriction since only one isolated particle in an infinite matrix is considered, i.e. $\nu \rightarrow 0$ (e.g. [7]). In this paper, however, Equation 18 is used which describes the decrease of the critical stress dependent on the residual stresses as well as on the volume fraction ν .

3.2. Calculations of dissipated and stored energy

Now attention is focused on the microcracks formed in the matrix. Let us assume that a large number of microcracks exist which are evenly dispersed in the matrix. Then the matrix, although containing microcracks, can be considered as a continuum described by effective elastic properties

different from those of the uncracked matrix. This method allows us to calculate the specific dissipated energy η_D as well as the change in stored energy ($\eta_s - \eta_{s0}$) without looking into the details of microcrack propagation. To this end two stages are distinguished:

- (1) Before microcracking the matrix is homogeneous and its Young's modulus is equal to the composite modulus (cf. Equation 14)

$$E^m = E_0;$$

- (2) After microcracking the matrix is effectively homogeneous. The existence of microcracks, however, causes a decrease in Young's modulus depending on the density of microcracks (cf. Equation 16)

$$E^m = E = E(\rho) < E_0.$$

In order to calculate the function $E(\rho)$, the self-consistent scheme, widely known in the theory of effective elastic constants, seems applicable because it can be assumed that a random spatial distribution of microcracks is realized. As usual, the self-consistent scheme results in implicit equations. With regard to Young's modulus, however, the results can be very satisfactorily approximated by

$$\frac{E}{E_0} = 1 - k\rho \quad (21)$$

where k is a number which depends on the shape and orientation of the microcracks. Budiansky and O'Connell [26] applied the self-consistent scheme to a material containing microcracks. Their results for the case of randomly oriented penny-shaped cracks may be depicted by $k = 2/9$. The present authors [16] have investigated a two-dimensional array of plane parallel microcracks ($k = \pi/2$).

In this paper, penny-shaped cracks are considered which are oriented preferably perpendicular to the applied stress, i.e. parallel to the macrocrack (cf. Fig. 1).^{*} A slight modification of the procedures in [16] or [26] yields

$$k = 2/3. \quad (22)$$

Now the dissipated energy is calculated. To this end let us look at the stress-strain diagram of the matrix (Fig. 2). Prior to microcracking, loading is governed by Young's modulus E_0 (Stage 1). After microcracking the material possesses the modulus

^{*}Obviously the matrix becomes anisotropic since there is a preferred orientation of microcracks. In this case Equation 21 gives the modulus in the direction perpendicular to the microcracks (i.e. the direction of applied stress). Since the behaviour concerning this direction seems to be of major importance, the anisotropy is neglected.

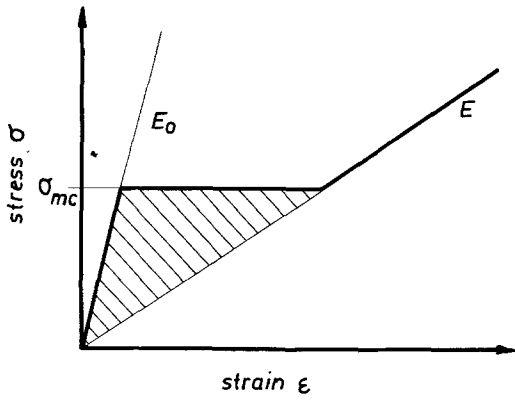


Figure 2 Stress-strain diagram of a material showing microcracking.

$E(\rho)$ (Stage 2). The transition between these two stages starts when the critical stress σ_{mc} is reached. The particular transition curve, however, will depend on the special features of the stochastic process of microcrack formation. From a detailed model of this stochastic process [24] it has been shown that a good approximation is provided by the assumption that the transition proceeds at constant stress (in a previous work [17] a sudden change in Young's modulus was assumed to occur, which seems, however, less realistic). Hence, the dissipated energy is given by the shaded area in Fig. 2 (considering that unloading is governed by E only)

$$\eta_D = (1-v) \left(\frac{\sigma_{mc}^2}{2E_0} \right) \left[\frac{(1-E/E_0)}{E/E_0} \right]. \quad (23)$$

The factor $(1-v)$, i.e. the volume fraction of the matrix, arises because η_D is defined as the dissipated energy per unit sample volume whereas Fig. 2 yields the energy per unit matrix volume.

It is noted that η_D given by Equation 23 comprises not only the energy converted into fracture surface energy of microcracks but also the dynamic energy emitted as elastic waves. This is obvious since the ratio E/E_0 has been calculated from the loss of total potential energy of the material due to microcracking. Thus the same holds for η_D . As it is known, only at the beginning of crack extension is the loss of potential energy equal to the fracture surface work, otherwise the former energy exceeds the latter.

Let us turn to the stored energy η_s , i.e. the elastic strain energy contained in the material in an unloaded state. At first a simple expression is derived which generally holds for a two-phase

composite. The specific stored energy is given by the volume average of the local strain energy

$$\eta_s = \overline{\frac{1}{2} \sigma_{ij} (\epsilon_{ij} - \epsilon_{ij}^T)} = \frac{1}{2} \overline{\sigma_{ij} \epsilon_{ij}} - \frac{1}{2} \overline{\sigma_{ij} \epsilon_{ij}^T}, \quad (24)$$

where ϵ_{ij} and ϵ_{ij}^T denote the total strain and the stress-free strain. The first term on the right-hand side of Equation 24 can be transformed into a surface integral which vanishes since the surface tractions are zero; the second term may be simplified because ϵ_{ij}^T is a given constant being equal either to $(\epsilon_{ij}^T)^p$ or $(\epsilon_{ij}^T)^m$:

$$\eta_s = -\frac{1}{2} [v \bar{\sigma}_{ij}^p (\epsilon_{ij}^T)^p + (1-v) \bar{\sigma}_{ij}^m (\epsilon_{ij}^T)^m]. \quad (25)$$

Here, $\bar{\sigma}_{ij}^p$ and $\bar{\sigma}_{ij}^m$ denote the volume averages with respect to the particles and the matrix. Since the average stress $\bar{\sigma}_{ij}$ vanishes, the following equation holds

$$\bar{\sigma}_{ij} = v \bar{\sigma}_{ij}^p + (1-v) \bar{\sigma}_{ij}^m = 0. \quad (26)$$

Making use of Equation 26, $\bar{\sigma}_{ij}^p$ can be eliminated from Equation 25 to obtain

$$\begin{aligned} \eta_s &= \frac{1}{2} (1-v) \bar{\sigma}_{ij}^m [(\epsilon_{ij}^T)^p - (\epsilon_{ij}^T)^m] \\ &= \frac{1}{2} (1-v) \bar{\sigma}_{ij}^m \Delta \epsilon_{ij}^T. \end{aligned} \quad (27)$$

This equation quite generally holds for a two-phase composite. Thus it is seen that it is sufficient to know only the average stress in one component of the composite in order to calculate the energy stored by residual stresses.

In our case the average matrix stress, derived from the composite-sphere model, is given by Equation 17. Introducing Equations 13 and 17 into Equation 27 yields (remember $\delta_{ij} \delta_{ij} = \delta_{ii} = 3$)

$$\eta_s = \frac{5}{2} v(1-v) \frac{E(\epsilon^T)^2}{(1+E/E_0) + v(1-E/E_0)}. \quad (28)$$

The energy η_{s0} , stored in the material before microcracking, may be easily calculated from Equation 28 if E is put equal to E_0 . In this way the result

$$\begin{aligned} (\eta_s - \eta_{s0}) &= -\frac{5}{2} \frac{\sigma_{mc}^2}{2E_0} v(1-v) \\ &\quad \times \left[\frac{(1-E/E_0)}{\left(1 + \frac{1-v}{1+v} E/E_0\right)} \right] \chi^2 \end{aligned} \quad (29)$$

is obtained where the parameter χ has been introduced, defined by Equation 19, which is a measure of residual stress. It is observed from

Equation 29 that the change of stored energy can be expressed by the ratio E/E_0 , i.e. the decrease of the modulus connected with microcracking (cf. Equation 21). Furthermore, $(\eta_s - \eta_{s0})$ is negative, which means that stored energy is released.

According to the general expression, Equation 10, the fracture toughness depends on the sum $(\eta_D + \eta_s - \eta_{s0})$. Hence, making use of Equations 23, 29, 21 and 22, the results may be summarized as follows

$$\eta_D + \eta_s - \eta_{s0} = \frac{\sigma_{mc}^2}{2E_0} g(\rho, v, \chi) \quad (30)$$

where the function g is given by

$$g(\rho, v, \chi) = (1-v) \frac{(1-E/E_0)}{E/E_0} \times \left[1 - \frac{5}{2}v \frac{E/E_0}{1 + \left(\frac{1-v}{1+v}\right)E/E_0} \chi^2 \right]$$

with $E/E_0 = 1 - \frac{2}{3}\rho$. (31)

This set of equations describes the joint action of energy dissipation and release of stored energy as a function of the density of microcracks, the volume fraction of particles and the residual stresses. A discussion of this result is postponed to Section 4.

3.3. Effective fracture surface energy

It may be seen from Equation 10 that there is only one unknown quantity left. This is the ratio γ/γ_0 , where γ denotes the effective fracture surface energy actually consumed in the process zone (i.e. the zone directly ahead of the crack tip) and γ_0 is the true fracture surface energy of the material which would be relevant if there were no structural changes resulting from microcracking.

Since the macrocrack may absorb a number of microcracks when it grows, only a reduced amount of fracture surface has to be created. Thus microcracking mainly results in a ratio $\gamma/\gamma_0 < 1$. On the other hand microcracks may cause branching, wandering or twisting of the crack front (cf. Wu *et al.* [27]) which increases γ . That means, even if the macrocrack path was known, it would be a very complicated geometrical problem to derive a simple formula for γ/γ_0 .

In addition to these geometrical problems we should contemplate that the extension of the macrocrack may be connected to release of stored

energy which is not comprised by $(\eta_s - \eta_{s0})$ in Equation 10 or 29. Although this additional amount of available energy may lead to a decrease in the effective fracture surface energy γ , it can be assumed that the main part of the stored energy has already been released when the dissipation zone has developed. Hence the remaining part of stored energy which might be released by the propagating macrocrack in the process zone is neglected.

The present paper is restricted to the case of the effective fracture surface energy γ being mainly dependent on the reduction of fracture surface to be created (as discussed above). Let $d\lambda$ denote that part of the total fracture surface which is contributed to (per unit thickness) by the microcracks when the macrocrack grows by a distance dl . The area $d\lambda$ requires no fracture work whereas the remaining part of the material consumes the specific work γ_0 . Therefore γ may be expressed by

$$\gamma = \gamma_0 \left(1 - \frac{d\lambda}{dl} \right). \quad (32)$$

Suppose that the process zone has a width $2p$, i.e. all the microcracks situated in a strip of width $2p$ are absorbed by the macrocrack. Then the number dN of microcracks absorbed per unit thickness when the macrocrack extends by dl is given by

$$dN = \mathcal{N} \cdot 2p \, dl = \frac{2p}{(2a)^3} \rho \, dl \quad (33)$$

where \mathcal{N} denotes the number of microcracks per unit volume (cf. Equation 11). Every microcrack contributes an amount πa^2 to the free surface $d\lambda$. Thus,

$$d\lambda = \pi a^2 \, d\mathcal{N} \quad (34)$$

Combining Equations 32, 33 and 34 one finally obtains

$$\gamma/\gamma_0 = 1 - \beta\rho \quad (35)$$

with

$$\beta = \frac{\pi p}{4 a}. \quad (36)$$

Equation 35 enables us to calculate the ratio γ/γ_0 dependent on the density ρ of microcracks. The width $2p$ of the process zone may be measured by the roughness of the fracture surface. p should be expected to be of the order of a . Therefore the range of possible β -values can be estimated as

$$0 \leq \beta \leq 2. \quad (37)$$

In the present paper β is regarded as a parameter

which assumes characteristic values for different classes of materials. For example, materials with a laminated structure may show low values of β . The influence of β on the macroscopic fracture toughness is discussed in the next Section.

4. Results and discussion

In order to summarize the results Equation 30 is introduced into the general expression, Equation 10. Thus we obtain the normalized fracture toughness

$$\frac{G_c}{2\gamma_0} = \frac{\gamma/\gamma_0}{1 - \frac{1}{3} \left[\frac{g(\rho, v, \chi)}{(\sigma_c/\sigma_{mc})^2} \right]}. \quad (38)$$

Three functions, having been calculated in the preceding section, enter into this relation: the ratio of fracture surface energies γ/γ_0 , the function g representing the joint action of energy dissipation and release of stored energy, and the ratio σ_c/σ_{mc} describing the decrease of the critical stress caused by residual stresses. These functions are given by Equations 35, 31 and 18 respectively. Thus the fracture toughness is a function of the density ρ of microcracks (created in the dissipation zone), the volume fraction v of particles and the residual stress χ . To illustrate the results let us first consider a single-phase ceramic with negligible residual stresses ($v = 0, \chi = 0$). Suppose that stable microcracking is possible. In this case Equations 31 and 18 reduce to

$$g = g(\rho, 0, 0) = \left(\frac{1 - E/E_0}{E/E_0} \right)$$

with

$$E/E_0 = (1 - \frac{2}{3}\rho) \quad (39)$$

and

$$\frac{\sigma_c}{\sigma_{mc}} = 1.$$

The resulting fracture toughness is plotted in Fig. 3 for different values of the process zone parameter β .* The curves show that an increase in G_c can be expected for low values of β and a sufficiently high microcrack density ρ . As discussed above this case could be realized in materials with a laminated structure. Normally, however, it is a very complicated task to obtain a sufficiently high microcrack density because the matrix may become unstable if ρ exceeds a certain critical value ρ_c .

Contrary to the Griffith-type instability of the macrocrack, instability of the matrix simply means

that the microcracks coalesce and large cracks are formed. This implies $\gamma \rightarrow 0$ and, accordingly, $G_c \rightarrow 0$ (such a critical value ρ_c is frequently called the "percolation point"). An analytical description of the percolation process is not yet available. Henceforth ρ_c is seen as a material-dependent constant. For $\rho < \rho_c$ Equation 38 holds whereas for $\rho > \rho_c$ a drop in the fracture toughness may be expected.

To conclude the discussion of Fig. 3, it is noted that there is another restriction on Equation 38 since, apparently, the toughness becomes infinite at a certain microcrack density; this is not a real result since the assumption of a small dissipation zone is violated if G_c becomes too large (cf. Equation 8). Hence we may state that the theory yields quantitative conditions which must be met to increase the fracture toughness by microcracking, yet a precise prediction of the exact value of the toughness should not be expected.

We now turn to the case of existing residual stresses, i.e. $\chi = E_0 \epsilon^T / \sigma_{mc} > 0$. Although the value of g in Equation 38 is lowered by the release of stored energy, the simultaneous decrease of σ_c/σ_{mc} , leading to a large dissipation zone, overcompensates for this. Thus, residual stresses result in an increase in the fracture toughness. Fig. 4 presents numerical results for the toughness dependent on ρ and χ (the volume fraction v and the process zone parameter β are kept constant). The curves in Fig. 4 show that the increase in G_c is shifted to lower values of the microcrack density ρ if χ is raised. Moreover, the influence of β is much less pronounced than in Fig. 3. Therefore it seems easier to realize a sufficiently high microcrack density without exceeding the critical density ρ_c . There is, however, an upper limit for the residual stress which may be estimated from Equation 18

$$\chi < \frac{12}{5} \left(\frac{1}{1 + 2v} \right). \quad (40)$$

If χ exceeds this limit, the critical stress σ_c becomes zero. This fact simply implies that microcracking occurs before loading starts, thus an increase of the fracture toughness by microcracking is impossible. In this way the theory shows that the residual stresses must be carefully adjusted by an appropriate choice of the components of the ceramic composite.

*These results are essentially the same as those published in a previous work [17]. There are only slight differences since, in that paper, we used a two-dimensional model instead of the penny-shaped cracks considered here. Additionally, in [17], an attempt was made to analytically describe the coalescence of microcracks.

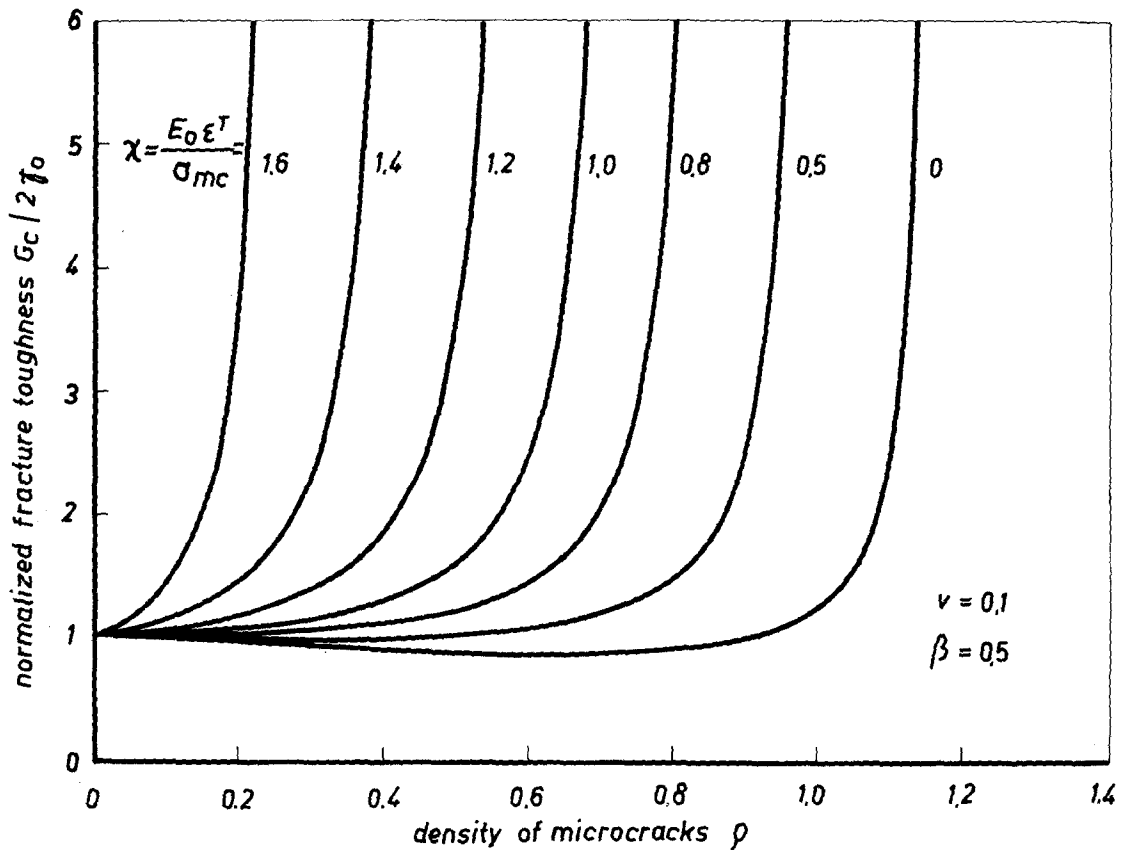


Figure 4 Fracture toughness as a function of microcrack density for varying residual stresses.

been plotted

$$\frac{K_c}{K_{c0}} = \left(\frac{G_c}{2\gamma_0} \right)^{\frac{1}{2}} \quad (42)$$

The constant f (cf. Equation 41) has been fitted to the experimental results of Claussen [8] which are also shown in Fig. 5; f -values have been obtained as depicted in Fig. 5. Expectedly, f depends on the diameter D of the zirconia particles. The resulting relationship between f and D can be approximately described by the relation $f \sim D^{3/2}$ which is equivalent to $a^2 \sim D^3$. At this point more detailed investigations are needed; this will be the subject of future work.

Fig. 5 shows that the increase of toughness is predicted fairly well by our theory. Above a certain volume fraction, however, the fracture toughness decreases suddenly. This may be explained by the assumption of a critical microcrack density ρ_c as discussed in connection with Fig. 3. According to these considerations the plot in Fig. 5 contains the theoretical curve for the fracture toughness depending on the volume fraction with $\rho = \rho_c$ being

kept constant. The value $\rho_c = 0.48$, independent of the particle size, yields the best correspondence between the theoretical and experimental results. Thus the maximum fracture toughness can also be predicted by theory.

It has been the main objective of this paper to understand how the fracture toughness can be raised by utilizing energy dissipative mechanisms and which role is played by the different microstructural parameters such as microcrack density, residual stresses and so on. It has been shown that theoretical considerations may be successfully applied in order to derive the conditions which must be met if the fracture toughness is to be improved. There are, of course, some unsolved problems. On what the microcrack density, ρ_c , depends and in which way ρ_c can be changed are both important questions since a small ρ_c value results in poor values of the toughness and the energy-dissipative mechanism cannot be utilized.

There is another condition that must be met if the fracture toughness is to be increased by energy

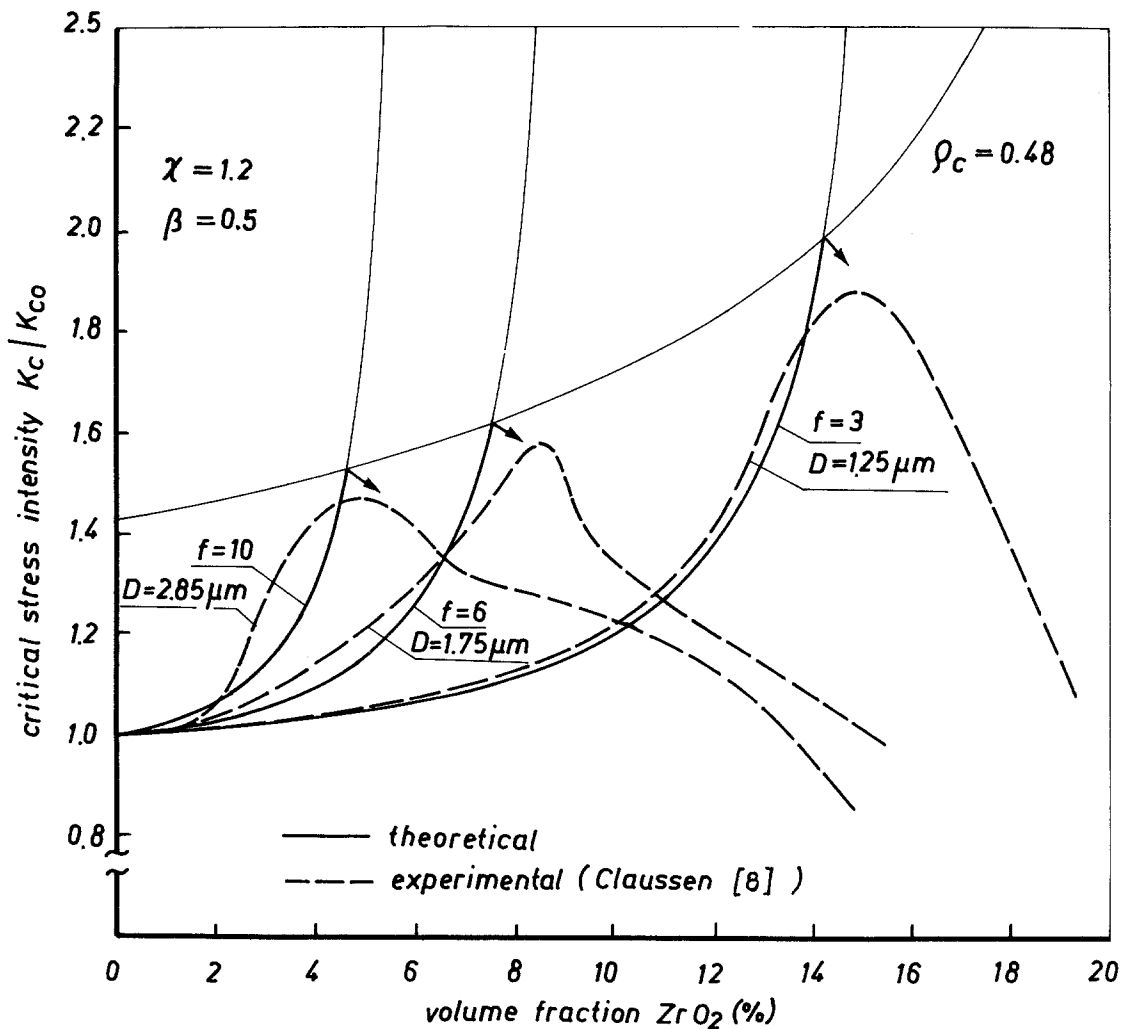


Figure 5 Comparison between theoretical and experimental results for a ceramic system consisting of an alumina matrix with zirconia particles embedded in it.

dissipation. Namely, the dissipation zone must be sufficiently large. Apparently, our final results do not depend on the absolute value of r_D . However, in deriving the theoretical relations, it has been assumed that the dissipation zone contains a large number of microcracks and that there is a clear distinction between the dissipation zone and the process zone. The validity of this assumption with respect to a particular material may be checked using Equation 8. If r_D is of the order of the microcrack size a , then our model is not applicable and a more detailed investigation is necessary.

For example let us consider the strength of such ceramics. As usual the largest flaw existing inside the material will determine the fracture stress. If this flaw is very small, then r_D will be of the

same size and the model is not applicable. On the contrary, if the strength is governed by large flaws, the energy-dissipative mechanism can be working, and strength increases with toughness. Thus it has turned out that strength and fracture toughness do not necessarily show the same behaviour.

In summary it may be stated that the theory enables the derivation of conditions which lead to an increase of the fracture toughness. The influence of residual stresses has especially been discussed. Although we have restricted ourselves to the case of energy dissipation by microcracking, other energy-dissipative mechanisms like the stress-induced phase transformation may be dealt with along the same lines. This will be a subject of future work.

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